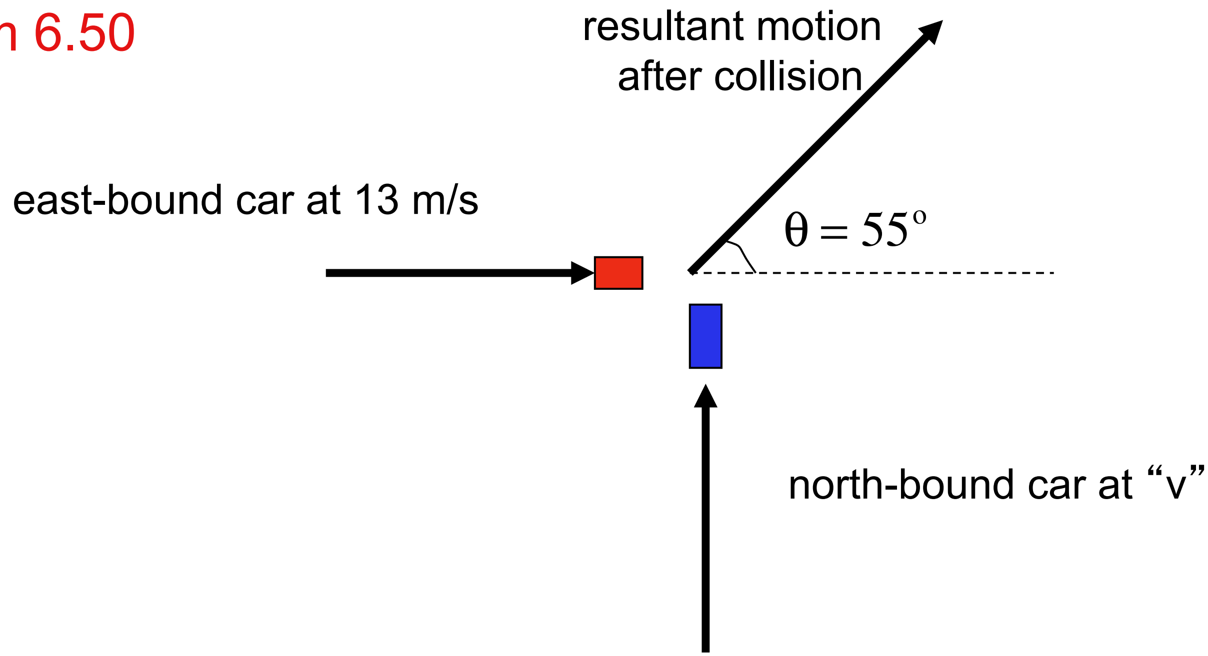
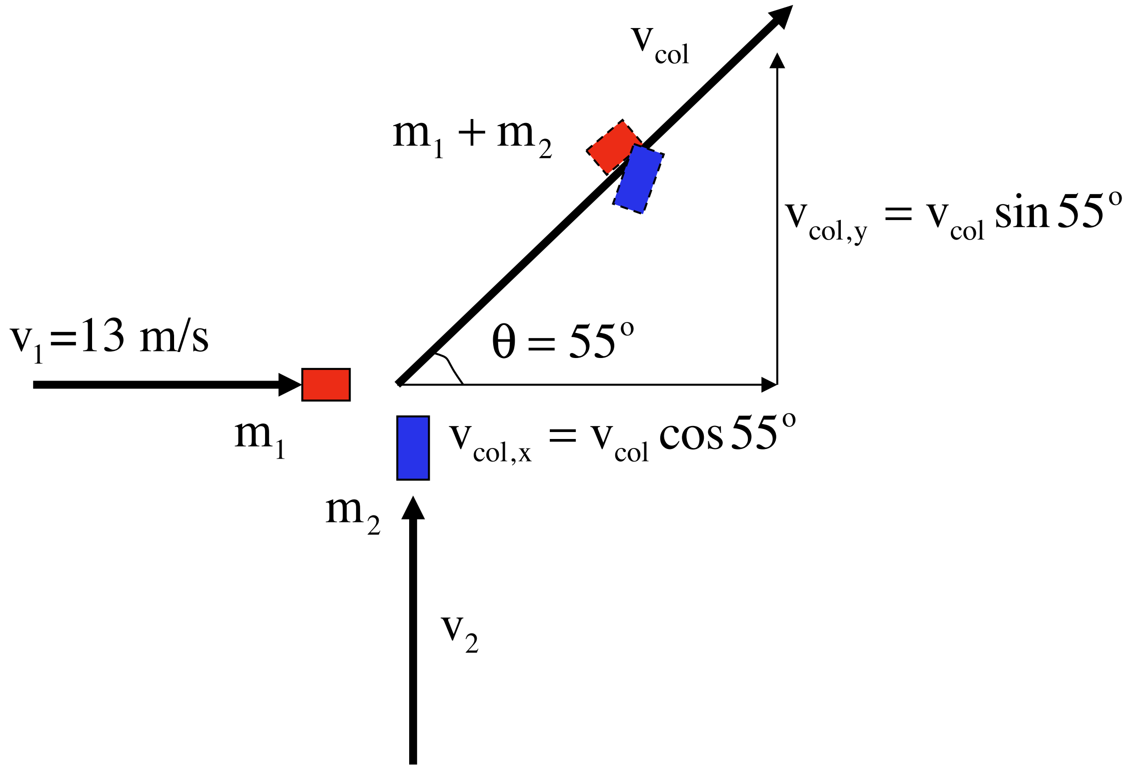
Problem 6.50



In this problem, you have to deal with the vector nature of momentum. In fact, you have to write out the conservation of momentum as it pertains to both the "x" and "y" directions.

Note: In this problem, we will have two unknowns--the north-bound car's velocity and the velocity of the two cars directly after the collision. As such, you can expect to need two equations to solve this problem.



let $m_1 = m_2 = m$ (the car masses are the same)

2.)

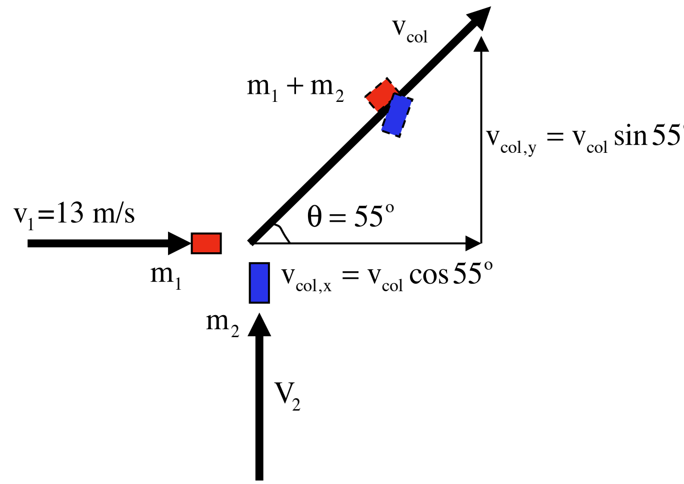
Conservation of momentum in the "x" direction yields:

$$\sum p_{1,x} + \sum F_{ext,x}\Delta t = \sum p_{2,x}$$

$$\Rightarrow mv_1 + 0 = (m+m)v_{col}\cos\theta$$

$$\Rightarrow (13\text{ m/s}) = (2)v_{col}\cos 55^o$$

$$\Rightarrow v_{col} = 11.33\text{ m/s}$$



Conservation of momentum in the "y" direction yields:

$$\sum p_{1,y} + \sum F_{ext,y}\Delta t = \sum p_{2,y}$$

$$\Rightarrow mv_2 + 0 = (m+m)v_{col}\sin\theta$$

$$\Rightarrow v_1 = (2)v_{col}\sin 55^o$$

$$=1.64v_{col}$$

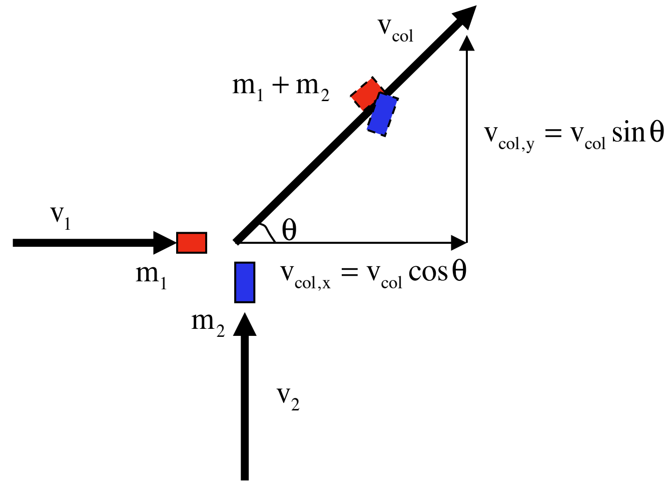
$$=1.64(11.33\text{ m/s})$$

$$=18.6\text{ m/s.}$$

WITH A TWIST: Let's assume we know the pre-crash speeds for both cars and want the after crash speed of both together along with the angle of departure. Assuming the masses are NOT the same:

Conservation of momentum in the "x" direction yields:

$$\sum p_{1,x} + \sum F_{ext,x}\Delta t = \sum p_{2,x}$$
$$\Rightarrow m_1 v_1 + 0 = (m_1 + m_2) v_{col} \cos\theta$$



Conservation of momentum in the "y" direction yields:

$$\sum p_{1,y} + \sum F_{ext,y}\Delta t = \sum p_{2,y}$$
$$\Rightarrow m_2 v_2 + 0 = (m_1 + m_2) v_{col} \sin\theta$$

To solve for the angle (so you can then put the angle back into either of the original equations and get the collision velocity), dividing the bottom equation into the top and take the inverse tangent of the ratio. That yields:

$$\frac{m_1 v_1}{m_2 v_2} = \frac{(m_1 + m_2) v_{col} \sin\theta}{(m_1 + m_2) v_{col} \cos\theta} \qquad \Rightarrow \qquad \frac{m_1 v_1}{m_2 v_2} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$